**KNT/KW/16/5057**

**Bachelor of Science (B.Sc.) Semester—I (C.B.S.) Examination**

**STATISTICS**

**Compulsory Paper—1**

**(Probability Theory)**

Time : Three Hours] [Maximum Marks : 50

**Note :—** All the **FIVE** questions are compulsory and carry equal marks.

1. (A) Explain classical approach and relative frequency approach to probability. State their merits and demerits.

   (B) State additive law of probability for n events. Prove it for 3 events 5+5

**OR**

   (E) Define :

   (i) A random experiment

   (ii) The sample space

   (iii) An event

   and give an example of each.

   (F) Give axiomatic definition of probability and prove the following :

   (i) $P(\phi) = 0$

   (ii) $P(\overline{A}) = 1 - P(A)$

   (iii) If $A \subset B$ then $P(A) \leq P(B)$. 5+5

2. (A) Define conditional probability. Show that it satisfies the axioms of probability. If $P(\overline{A} \cap B) = 0.1$, $P(A \cap \overline{B}) = 0.6$ and $P(A \cap B) = 0.2$ find $P(A \mid B)$.

   (B) State and prove multiplicative law of probability, for n events. 5+5

**OR**

(E) Define mutual independence of n events. Show that the number of conditions to be satisfied for mutual independence of n events is $2^n-n-1$.

(F) Two unbiased dice are thrown. Let A denote the event of an odd total, B the event of getting a six on the first die and C the event of a total of seven. Are A, B and C mutually independent ? 5+5

3. (A) Define a random variable. Let X denote the product of number of heads obtained and the number of tails obtained when three unbiased coins are tossed. Find :

(i) The p.m.f. of X

(ii) E(X)

(iii) V(X).

(B) Define mathematical expectation of :

(i) a r.v. X

(ii) a function of a r.v. X.

State and prove properties of mathematical expectation. 5+5

**OR**

(E) Define a continuous r.v. Let X be a continuous r.v. with p.d.f. $f(x) = cx^2, 0 < x < 1$

$= 0$ elsewhere

(i) Determine C

(ii) Find E(X)

(iii) Find $P[X > 0.5]$.

(F) The diameter of an electric cable, say X, is a continuous r.v. with p.d.f.

$f(x) = 6x(1 - x), 0 \leq x \leq 1.$

$= 0$ elsewhere

(i) Obtain c.d.f. of X

(ii) Determine the number b such that

$P[X < b] = 2P[X > b]$

(iii) Compute $P[X \le 1/2 \mid 1/3 < X < 2/3]$. 5+5

4. (A) Define :

(i) $r^{th}$ raw moment

(ii) $r^{th}$ central moment

of r.v. X.

Derive the expression for $r^{th}$ central moment in terms of raw moments. Hence obtain the expressions for second, third and fourth central moments in terms of raw moments. 10

**OR**

(E) Define the following measures of location for a probability distribution :

(i) Arithmetic mean

(ii) Median

(iii) Mode

If X is a r.v. with p.m.f. $p(x)=\frac{x+1}{15}$, x = 0, 1, 2, 3, 4. Find the Mean, Mode and Median of X. 10

5. Solve any **TEN** of the following :

(A) Let A, B and C be any three events. Write expressions for the events :

(i) At least one of the three occurs

(ii) Only A occurs.

(B) Two cards are drawn randomly from a pack of 52 cards. Find the probability that both are aces.

(C) Three coins are tossed. Write the sample space and find the probability of getting two heads.

(D) An unbiased die is thrown. Find the probability of getting an even number given that the number obtained is greater than two.

(E) State Chebyshev's inequality.

(F) If $P(A \cup B) = 0.8$, $P(B) = 0.4$ find $P(\overline{A} \mid \overline{B})$.

(G) Let X be a r.v. with p.m.f. given below :

| **x** | **P[X = x]** |
|---|---|
| 1 | a |
| 2 | 2a |
| 3 | 3a |

Determine a.

(H) Let X be r.v. with p.d.f. $f(x) = 4x^3$, $0 < x < 1$. Find E(x).

(I) Let X be a r.v. with E(X) = 3 and V(X) = 9. Find :

(i) E(3x + 1)

(ii) V(3x + 1).

(J) Define standard deviation of a random variable.

(K) If the p.g.f. of a random variable X is $P_x(s) = \left(\frac{1}{2} + \frac{1}{2}s\right)^8$. Find E(x).

(L) Define a measure of Kurtosis. 1×10=10